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LETTER TO THE EDITOR

Dynamics of surface roughening in disordered media

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Abstract. We present results on the roughening of growing interfaces obtained from a Kardar–Parisi–Zhang (KPZ) type continuum equation with quenched additive noise, representing frozen in disorder. Close to the pinning transition, for the exponents describing respectively the temporal and the spatial scaling of the surface from numerical integration in $1+1$ dimensions we obtain $\beta = 0.61 \pm 0.06$ and $\alpha = 0.71 \pm 0.08$ up to a crossover time. These estimates are in good agreement with the theoretical prediction $\beta = \frac{3}{5}$ and $\alpha = \frac{3}{4}$ we derive from a dimensional analysis of the equation.

The roughening of growing interfaces subject to random perturbations is a very common phenomenon in nature and technologies [1–3]. There are two major types of this kind of growth: roughening (i) in the presence of temporally uncorrelated fluctuations and (ii) in disordered media, representing quenched noise. The first case is theoretically better understood due to much recent interest in the Kardar–Paris–Zhang (KPZ) [4] and closely related equations. Less is known about the dynamics of interfaces whose motion is dominated by the pinning forces present in an inhomogeneous medium. Since this case is relevant in many experimental situations such as two phase fluid flows in porous media [5–8], the motion of domain walls in magnetically ordered systems [9] or the pinning of charge density waves [10, 11], it is of great importance to understand the dynamics of such growing surfaces.

Studies of kinetic roughening with quenched noise have been concentrating on the two exponents α and β describing respectively the static and time dependent scaling of the surface width in the pinning dominated regime. The few very recent results for these exponents obtained from experiments, theory and simulations have not led to a consistent picture yet. In the most studied $(1+1)$-dimensional case the *experiments* on viscous flows gave estimates $0.63 < \alpha < 0.81$ [5, 6] and $\beta \simeq 0.65$ [6] and $\beta \simeq 0.45$ [8]. The *theoretical* value for the exponent β was suggested by Parisi [11] and Nattermann *et al* [12] to be $\frac{3}{4}$. In a recent preprint Kaganovich [13] (using a power counting type approach of Hentschel and Family [14]) proposed that the exponents corresponding to the KPZ equation with quenched noise are $\beta = 0.6$ and $\alpha = 0.75$. Finally, the *simulation* of various growth models and model equations resulted in diverse values as well. Two closely related growth models [15, 16] which were interpreted in terms of directed percolation yielded α and β both close to 0.63. $\beta \simeq 0.75$ was obtained from the model of Parisi [11]. The numerical integration of an Edwards–Wilkinson [17] type continuum

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equation with additive quenched noise led to the conclusion that α crosses over from 1 to 0.5 as the velocity of the surface is increased [18]. On the other hand, from the numerical study of an equation with quenched multiplicative noise a robust scaling in time with an exponent $\beta \simeq 0.65$ was found [19]. One of the important open questions is the dynamics of the quenched version of the KPZ equation which, as such, has not yet been numerically investigated. The above results are assumed to describe a growth regime dominated by pinning forces which can completely stop the moving interface in an important limiting case called pinning transition.

In this letter we present the first simulation results on the temporal scaling (β) of growing interfaces obtained from a KPZ-type continuum equation in 1+1-dimensions with quenched additive noise, representing frozen in disorder. We find a rich behaviour which is, in part, consistent with some of the recent experimental and simulational results. Close to the pinning transition $\beta \simeq 0.61$ and $\alpha \simeq 0.71$ are obtained in very good agreement with our dimensional analysis of the equation yielding $\beta = (4-d)/(4+d) = 3/5$ and $\alpha = (4-d)/4$, where d is the dimension of the initially flat surface.

To describe the roughening of a moving interface in random media we consider the quenched version of the KPZ equation

$$\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + v + \eta(\boldsymbol{r}, h) \tag{1}$$

where h is the position of the surface above the point $\boldsymbol{r}$ of the d-dimensional substrate at time t, ν and λ are constants. In (1) $\langle \eta(\boldsymbol{r}, h) \rangle = 0$ represents quenched noise whose correlator in the continuum limit is formally given by $\langle \eta(\boldsymbol{r}_0, h_0)\eta(\boldsymbol{r}_0 + \boldsymbol{r}, h_0 + h') \rangle = D\delta(h')\delta^d(\boldsymbol{r})$, where D is a constant. As we shall see, in the simulations and the theoretical interpretation of the numerical results we have to consider that the discretization of equation (1) corresponds to a somewhat modified form for the noise correlations.

The control parameter in this equation is the driving force v; above a critical v_c the interface keeps moving for arbitrary large times, however, if $v < v_c$ the interface after some characteristic time becomes pinned, i.e. due to the randomly distributed negative values of η it stops completely. For $\lambda = 0$ (1) is equivalent to the Edwards–Wilkinson equation with quenched disorder. This latter equation has been studied more intensively, in part, because it has been assumed that the nonlinear term of the KPZ type equation is automatically generated in the EW version by the noise which depends on h in a highly nonlinear way. Since our dimensional analysis indicated that the two equations may exhibit different scaling behaviour we decided to simulate the nonlinear equation directly.

The numerical integration of (1) for the (1+1)-dimensional case was carried out using a simple single step method (see, e.g. [20]). The discretized version of (1) is

$$h(x, t+\Delta t) = h(x, t) + \Delta t\{h(x-1, t) - 2h(x, t) + h(x+1, t)\}$$
$$+ \Delta t \left\{ \frac{\lambda}{2}(h(x+1, t) - h(x-1, t))^2 + v + \eta(x, [h(x, t)]) \right\}$$

where $[\cdot]$ denotes integer part, and η was chosen to be uniformly distributed on the interval $(-\frac{1}{2}, \frac{1}{2})$. We assumed that as the surface was moving between two grid points in the vertical direction, it experienced the effect of the fixed value of η associated with the closest grid point below the surface. Obviously, this does not correspond to a $\delta(h')$ kind of correlations, on the other hand, this is the only reasonable choice in the framework of the given method. In this respect the lattice spacing a ($a=1$ in the simulations) represents the characteristic length (of density correlations) in the inhomogeneous medium.

In order to produce simulations for long times we used $\Delta t=0.1$, and $\Delta t=0.01$. We checked our choice for Δt by making runs for smaller Δt and comparing the results. In most of the cases we used system sizes $L=1300$ and $l=5000$ lattice points along the x direction, and $\lambda=1$.

The following quantities were determined: (i) the surface width $w(t)=(\langle h^2\rangle-\langle h\rangle^2)^{1/2}$ as a function of time, (ii) the average position of the interface $h(t)=\langle h(x,t)\rangle$, and (iii) the surface width $w(x)$ as function of x in the saturated regime (in which $w(t)\simeq\text{const}$).

In figure 1 we show snapshots of the growing surface near pinning. It can be clearly seen that after '*breaking away*' at a locally almost pinned point the surface grows isotropically in both directions.

For $\lambda=1$ we found that the pinning threshold was $v_c^*\approx 0.05$. Just above this value we observed a scaling regime of $w(t)\sim t^\beta$. The exponent β can be obtained by fitting

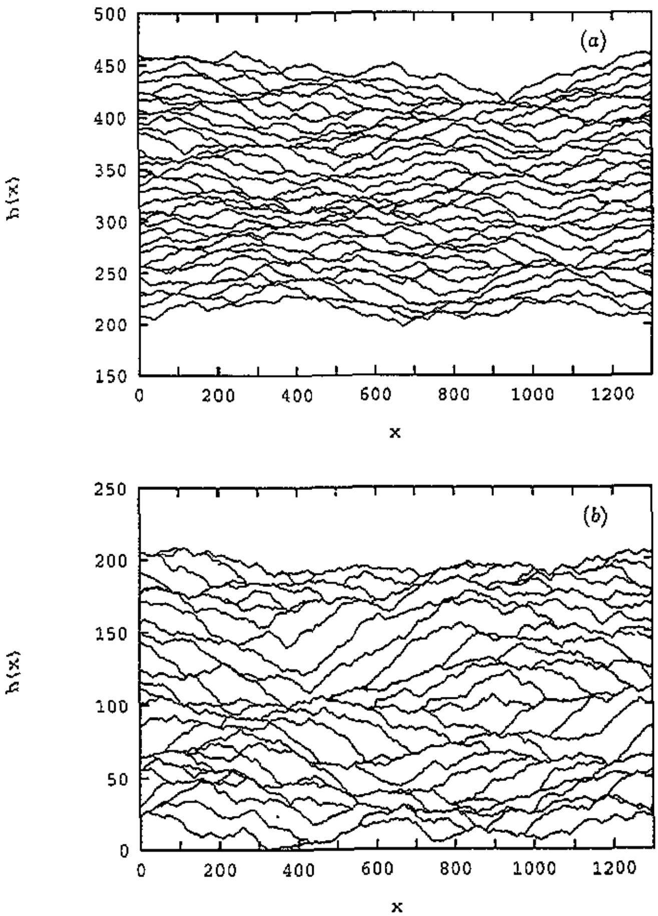
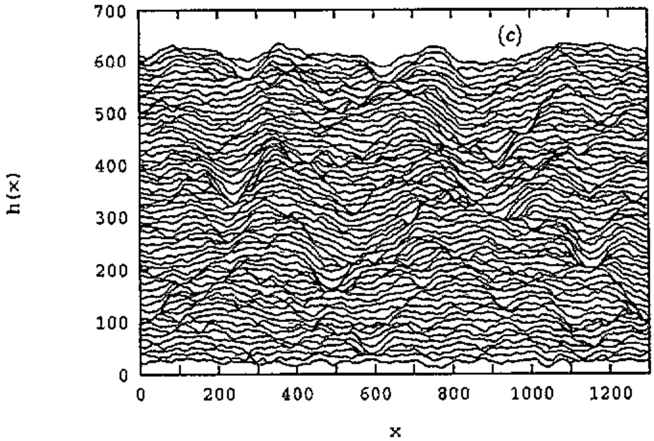


Figure 1. Subsequent 'snapshots' of the evolving surface for $L=1300$ obtained by numerically integrating equation (2). (a) KPZ case for $v=0.05$; (b) for $v=0.037\simeq v_c$; (c) EW case ($\lambda=0$) for $v=0.205$; (d) for $v=0.19\simeq v_c$.



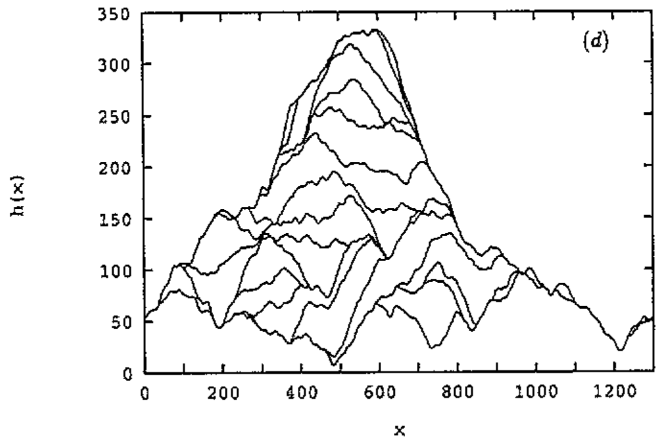


Figure 1. (continued)

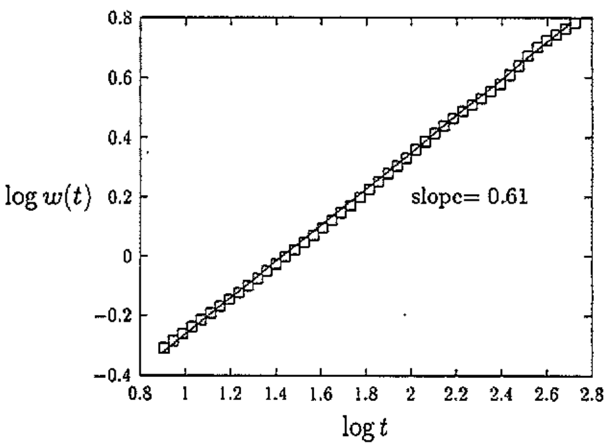


Figure 2. The time dependence of the surface width w in the quenched version of the KPZ equation for $L = 1300$ and $v = 0.04$.

a straight line on a log–log plot of $w(t)$ for $v \approx v_c$. From figure 2 we conclude that the dynamics of surface roughening is described by

$$\beta = 0.61 \pm 0.06$$

in the case of the discretized, quenched version of the KPZ equation. This value is *different* from that given in [11, 12].

A further important aspect of the growth is the geometry of the surface in the stationary regime. We found that *as the system size becomes larger* the behaviour of the surface approaches self-affine scaling with a static exponent close to $\alpha = 0.71$. Thus, our results suggest that this value may not be an effective exponent due to crossover [18], but the result of a genuine scaling regime. Figure 3 shows the corresponding results for $L = 5000$. If, for example, $L = 1300$ the scaling part of the plot is much shorter.

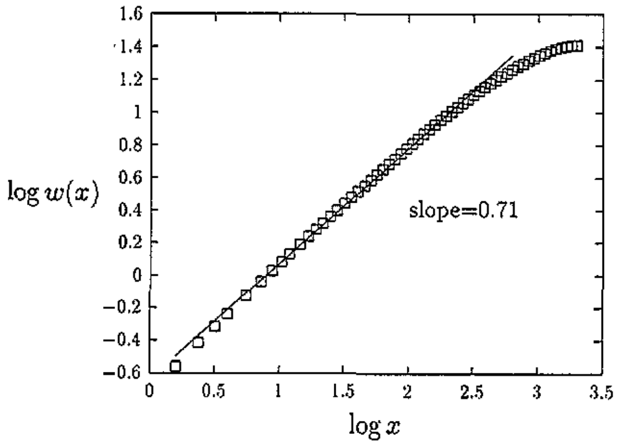


Figure 3. The behaviour of the surface width w (KPZ case) for $L = 5000$ and $v = 0.04$ as a function of the length x over which its average value was calculated. The middle part of the plot indicates self-affine scaling with an exponent $\alpha = 0.71$.

For $\lambda = 0$ (EW equation) we found that close, but above the pinning threshold ($v_c \simeq 0.19$) the dynamics of the interface roughening is described by an effective exponent 0.62 as well (see figure 4). However, if we get closer to v_c, for longer times a *continuously growing* effective exponent can be observed. This explains an earlier numerical estimate $\beta \simeq 0.75$ [11].

From the $h(t)$ data we calculated the asymptotic velocity $v_a = \lim_{t\to\infty} \partial h/\partial t$ for the EW case. It vanishes at $v = v_c$ and for $v > v_c$ it scales as $v_a \sim (v - v_c)^\theta$ [11, 12]. This allows us to extract the real pinning threshold v_c from simulation data. We did this *for $L = 1300$ by finding the v_c value providing the best fitting to the scaling relation.* We obtained $v_c = 0.183 \pm 0.005$ and $\theta = 0.64 \pm 0.08$ (see figure 5) in agreement with the prediction of Natterman *et al* $\theta = \frac{2}{3}$ for $d = 1$ [12].

In order to give a theoretical interpretation of the above numerical results we have carried out a detailed dimensional analysis of equation (1). To proceed we first need to point out that in real materials and the simulations the correlations describing the disorder are not delta function like. In particular, in the numerical studies the surface is discretized in the horizontal direction (along the substrate) and moves continuously in the vertical direction. Since the noise is discretized in both directions, as the surface advances, it experiences a correlated effect of the local value of η in the growth

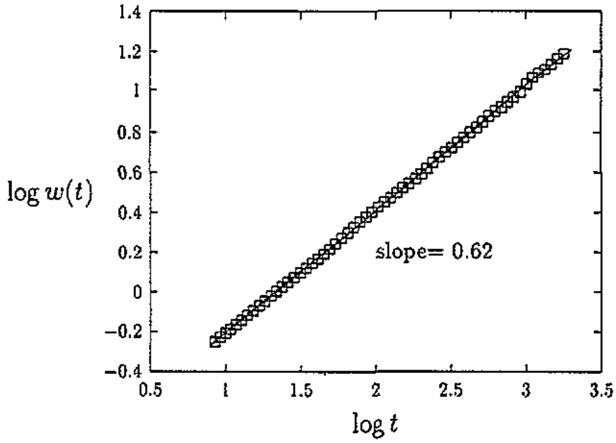


Figure 4. The temporal scaling of the surface width w in the quenched version of the EW equation for $L=1300$ and $v=0.205$.

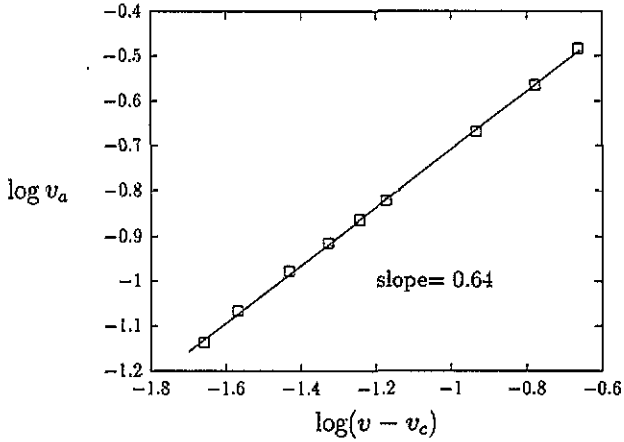


Figure 5. The dependence of the average velocity of the interface as a function of the difference from the critical driving velocity v_c corresponding to pinning. This plot was obtained for the EW case with $L=1300$.

direction. Correspondingly, we are led to study the effects of noise having the correlator (see also [12])

$$\langle \eta(r_0, h_0)\eta(r_0+r, h_0+h')\rangle = \Delta(|h'|)\delta^d(r). \tag{2}$$

In $\Delta(|h'|)$ is a monotonically decreasing function with a cutoff at a small characteristic size a playing the role of the typical correlation length of the fluctuations of noise in the h direction. Since h is the relevant direction from the point of growth, for the other directions we can keep the original assumption (δ) for the correlations.

By denoting the dimension of the corresponding quantities by $[h]$, $[t]$, $[r]$, $[\lambda]$, $[v]$, $[\nu]$, $[\eta]$ and $[\Delta]$ we get the following equations from (1) and (2): $[h]/[t]=[\nu][h]/[r]^2=[\lambda][h]^2/[r]^2=[v]=[\eta]=[\Delta]^{1/2}[r]^{-d/2}$. Thus, the quantities

$$H=\lambda^{-d/(4+d)}\Delta^{2/(4+d)}t^{(4-d)/(4+d)}$$

$$R=\lambda^{2/(4+d)}\Delta^{1/(4+d)}t^{4(4+d)}. \tag{3}$$

are such that $[H]=[h]$ and $[R]=[r]$. Using these relations we can also express $[v]$ and $[\nu]$ through $[\lambda]$, $[\Delta]$ and $[t]$. The dimension of the squared surface width $w^2(L, t)=\langle h^2\rangle-\langle h\rangle^2$ over a region of linear extent L is $[H]^2$. Therefore, the dimensionless width $w^2(L, t)/H^2$ has to be the function of the dimensionless variables L/R, $\tilde{v}/(H/t)$ and $\nu/(R^2/t)$, where $\tilde{v}=v-v_c$. After some trivial algebra (applied in order to eliminate t from the term corresponding to $\nu/(R^2/t)$) we get

$$w^2(L, t)=\lambda^{-2d/(4+d)}\Delta^{4/(4+d)}t^{2(4-d)/(4+d)}$$
$$\times f\left(\frac{L}{\lambda^{2/(4+d)}\Delta^{1/(4+d)}t^{4/(4+d)}} \quad \frac{\tilde{v}^{4+d}\lambda^d t^{2d}}{\Delta^2} \quad \frac{\nu^{2d}\tilde{v}^{4-d}}{\Delta^2\lambda^d}\right). \tag{4}$$

From the above expression it follows that for fixed $\nu^{2d}\tilde{v}^{4-d}/\Delta^2\lambda^d$ and $t \ll t^* = (\tilde{v}^{4+d}\lambda^d/\Delta^2)^{-1/2d}$ the scalings $w(L, t) \sim t^\beta$ and $w(L, t) \sim L^\alpha$ are satisfied with

$$\alpha = \frac{4-d}{4} \qquad \beta = \frac{4-d}{4+d}. \tag{5}$$

Above the crossover time t^* the standard KPZ type behaviour sets in. It should be noted that the temporal scaling of w^2 can be observed for t much smaller than the saturation time $t_s(L)$, while the spatial scaling holds for $t \gg t_s(L)$, where $t_s(L) = L^{(4+d)/4}/(\lambda^2\Delta)^{1/4}$. Therefore, $d_c=4$ is the upper critical dimension of the problem in agreement with the previous theories (see, e.g. [10–12]). However, our values for the α and β represent a universality class distinct from that of the standard KPZ equation and agree with a recent proposition by Kaganovich [13]. In particular, for the much studied $d=1$ case the exponents are

$$\alpha = \tfrac{3}{4} = 0.75 \qquad \beta = \tfrac{3}{5} = 0.6. \tag{6}$$

In conclusion, our results indicate that the quenched version of the KPZ equation represents a new universality class with a well defined scaling in time. The prediction $\beta = \frac{3}{5}$ is in good agreement with our estimate obtained from the numerical integration and is close to the related previous results $\beta \simeq 0.63$ [15, 16] and $\beta \simeq 0.65$ [6]. The above dimensional analysis can be pursued further to include crossover effects. This will be presented in a subsequent paper.

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